SANTOSH Academia
IIT-JEE | NEET | Foundation

# Answers & Solutions

Time : 3 hrs. M.M. : 300

*for*

# JEE (Main)-2025 Phase-1

# [Computer Based Test (CBT) mode]

## (Mathematics, Physics and Chemistry)

**22/01/2025**

**Morning**

**IMPORTANT INSTRUCTIONS:**

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (MPC) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three** Parts. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

# MATHEMATICS

SECTION - A

Multiple Choice Questions:This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is:

   (1) 4356 (2) 6084

   (3) 14950 (4) 5148

**Answer (4)**

**Sol.** _ _ M _ _

before M, after M

M is at 13th position

$\Rightarrow \left[{}^{12}C_2 \cdot (1)\right]_{\text{before M,}}[1]_{\text{M,}}\left[{}^{13}C_2\,(1)\right]_{\text{after M}}$ ways

= 5148

2. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2)$ is:

   (1) $18\pi^2$ (2) $31\pi^2$

   (3) $24\pi^2$ (4) $22\pi^2$

**Answer (4)**

**Sol.** Let $f(x) = 16[(\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2]$

$$f(x) = 16\left[(\sec^{-1}x + \text{cosec}^{-1}x)^2 - 2(\sec^{-1}x)\left(\frac{\pi}{2} - \sec^{-1}x\right)\right]$$

$$= 16\left[\frac{\pi^2}{4} - \pi\sec^{-1}x + 2\sec^{-1}x\right|_{\sec^{-1}x \in [0,\pi] - \left\{\frac{\pi}{2}\right\}}]$$

$$= 16\left[2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{4} - \frac{\pi^2}{8}\right|]$$

$f(x)$ minimum at $\sec^{-1}x = \frac{\pi}{4}$ and maximum at $\sec^{-1}x = \pi$

$\Rightarrow f(x)_{min} = 2\pi^2$, $f(x)_{max} = 20\pi^2$

3. The number of non-empty equivalence relations on the set {1, 2, 3} is:

   (1) 5 (2) 7

   (3) 4 (4) 6

**Answer (1)**

**Sol.** Number of non-empty equivalence relations on the set {1, 2, 3} will be bell number 3 = 5

{{1,} {2}, {3}},

{{1, 2}, {3}}, {{1, 3}, {2}, {2, 3}, {1}},

{{1, 2, 3}}. Therefore, total non-empty equivalence relation is 5.

4. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between $L_1$ and $L_2$?

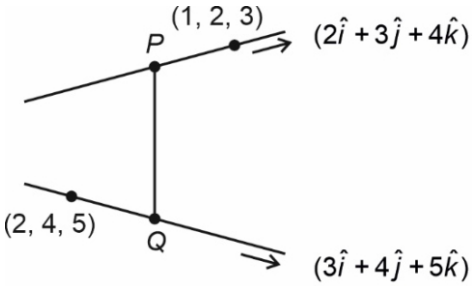
   (1) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (2) $\left(2, 3, \frac{1}{3}\right)$

   (3) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$ (4) $\left(-\frac{5}{3}, -7, 1\right)$

**Answer (3)**

**Sol.** $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$



$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$

Dr's of $PQ < 2\lambda - 3\mu - 1, 3\lambda - 4\mu - 2,$

$4\lambda - 5\mu - 2 >$

$$PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\Rightarrow \quad \frac{2\lambda - 3\mu - 1}{-1} = \frac{3\lambda - 4\mu - 2}{2} = \frac{4\lambda - 5\mu - 2}{-1}$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \mu = \frac{-1}{6}$$

$$\Rightarrow \quad P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$$

Dr's $PQ \langle 1, -2, 1 \rangle$

$\therefore$ Line

$$\frac{y - \frac{5}{3}}{1} = \frac{y - 3}{-2} = \frac{y - \frac{13}{3}}{1}$$

$\therefore$ Option (3) satisfies

5. Let the foci of a hyperbola be (1, 14) and $(1, -12)$. If it passes through the point (1, 6), then the length of its latus-rectum is:

(1) $\frac{25}{6}$ (2) $\frac{288}{5}$

(3) $\frac{144}{5}$ (4) $\frac{24}{5}$

**Answer (2)**

**Sol.** $|SP - S'P| = 2a$

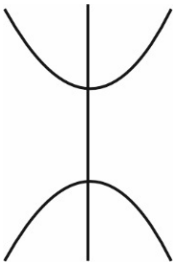
$SS' = 2ae$

$S(1, 14) \qquad S'(1, -12) \qquad P(1, 6)$

$2a = |8 - 18|$

$a = 5$

$2ae = 26 \Rightarrow ae = 13$



$$L(LR) = \frac{2b^2}{a} = \frac{2a^2(e^2 - 1)}{a}$$

$$= \frac{2[169 - 25]}{5} = \frac{288}{5}$$

6. A coin is tossed three times. Let $X$ denote the number of times a tail follows a head. If $\mu$ and $\sigma^2$ denote the mean and variance of $X$, then the value of $64(\mu + \sigma^2)$ is:

(1) 48 (2) 32

(3) 64 (4) 51

**Answer (1)**

**Sol.**

| | $x_i$ | $P_i$ |
|---|---|---|
| HHH | 0 | $\frac{1}{8}$ |
| TTT | 0 | $\frac{1}{8}$ |
| HHT | 1 | $\frac{1}{8}$ |
| HTH | 1 | $\frac{1}{8}$ |
| THH | 0 | $\frac{1}{8}$ |
| TTH | 0 | $\frac{1}{8}$ |
| THT | 1 | $\frac{1}{8}$ |
| HTT | 1 | $\frac{1}{8}$ |

$\mu = \sum x_i P_i = \frac{1}{2}$

$\sigma^2 = \sum x_i^2 P_i - \mu^2$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$64(\mu + \sigma^2) = 64\left[\frac{1}{2} + \frac{1}{4}\right]$

$= 64 \times \frac{3}{4} = 48$

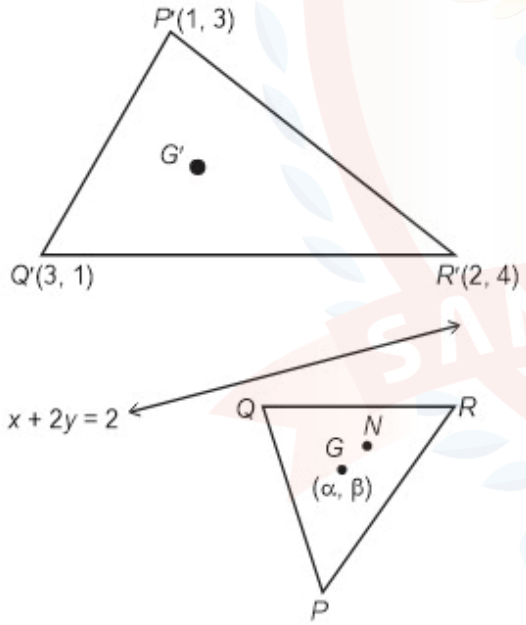
7. Let the triangle $PQR$ be the image of the triangle with vertices (1, 3), (3, 1) and (2, 4) in the line $x + 2y = 2$. If the centroid of $PQR$ is the point $(\alpha, \beta)$, then $15(\alpha - \beta)$ is equal to:

(1) 19 (2) 24

(3) 22 (4) 21

**Answer (3)**

**Sol.** The centroid $G''(\alpha, \beta)$ of $\Delta PQR$ be image of centroid of given triangle $P'Q'R'$.




Centroid of $P'Q'R' = \left(\frac{1+3+2}{3}, \frac{3+1+4}{3}\right) = G'\left(2, \frac{8}{3}\right)$.

Image of $G'\left(2, \frac{8}{3}\right)$, w.r.t. line $x + 2y = 2$ is $(\alpha, \beta)$

Then $\frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = \frac{-2\left(2 + \frac{16}{3} - 2\right)}{1+4}$

$\therefore \quad \frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = -\frac{32}{15}$

$\therefore \quad \alpha = -\frac{2}{15}$ and $\beta = -\frac{8}{5}$

Then $15(\alpha - \beta) = 15\left(-\frac{2}{15} + \frac{24}{15}\right) = 22$

8. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x) f'(y) + f'(x) f(y)$ for all $x, y \in R$.

Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to:

(1) 2406 (2) 2384

(3) 2525 (4) 5220

**Answer (3)**

**Sol.** $\because f(x + y) = f(x).f'(y) + f'(x).f(y), \forall x, y \in R$ ...(i)

And $f(0) = 1$ ...(ii)

Now replace $x$ by zero and $y$ by zero we get

$f(0) = f(0) f'(0) + f'(0) f(0)$

$1 = f'(0) + f'(0)$

$\therefore \quad f'(0) = \frac{1}{2}$ ...(iii)

Now replace $y$ by zero in equation (i), we get

$f(x) = \frac{1}{2} f(x) + f'(x)$

or, $\frac{1}{2} f(x) = f'(x)$

then $\frac{f'(x)}{f(x)} = \frac{1}{2}$

hence $\ln|f(x)| = \frac{x}{2} + c$

Put $x = 0$, we get $c = 0$

$\therefore \quad \ln|f(x)| = \frac{x}{2}$

Then $\sum_{n=1}^{100} \ln(f(\eta)) = \left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + ... + \frac{100}{2}\right)$

$= \frac{5050}{2} = 2525$

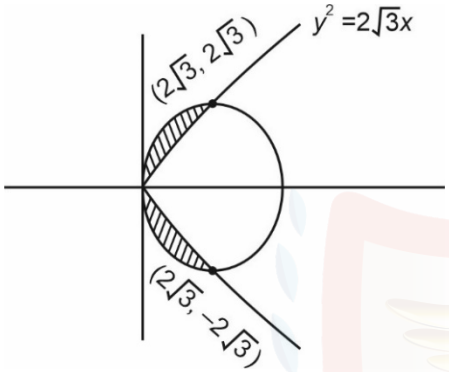
9. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is:

(1) $3\pi-8$ (2) $3\pi+8$

(3) $6\pi-16$ (4) $6\pi-8$

**Answer (3)**

**Sol.**




Required area $= 2\int_0^{2\sqrt{3}}\left(\sqrt{4\sqrt{3}x-x^2}-\sqrt{2\sqrt{3}x}\right)dx$

$= 2\int_0^{2\sqrt{3}}\left(\sqrt{12-\left(x-2\sqrt{3}\right)^2}-\sqrt{2\sqrt{3}x}\right)dx$

$= 2\left[\frac{x-2\sqrt{3}}{2}\sqrt{12-\left(x-2\sqrt{3}\right)^2}+\frac{12}{2}\sin^{-1}\left(\frac{x-2\sqrt{3}}{2\sqrt{3}}\right)-\frac{\sqrt{2\sqrt{3}}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{2\sqrt{3}}$

$= 2\{3\pi-8\}$

$= 6\pi-16$ sq. units.

10. Let $a_1$, $a_2$, $a_3$, .... be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2 + a_4 = 29$, then $a_6$ is equal to:

(1) 628 (2) 812

(3) 526 (4) 784

**Answer (4)**

**Sol.** $a_1a_5 = 28 \Rightarrow a^2r^4 = 28$

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$ar$ and $ar^3$ are roots of

$k^2 - 29k + 28 = 0$

$\Rightarrow \quad k = 1, k = 28$

$ar = 1, ar^3 = 28$

$r^2 = 28, a^2 = \frac{1}{28}$

$a_6 = ar^5 \Rightarrow a_6^2 = a^2r^{10} = (28)^4$

$\Rightarrow \quad a_6 = 784$

11. Let $z_1$, $z_2$ and $z_3$ be three complex numbers on the circle $|z| = 1$ with $\arg(z_1)=\frac{-\pi}{4}$, $\arg(z_2)=0$ and $\arg(z_3)=\frac{\pi}{4}$. If $\left|z_1\bar{z}_2+z_2\bar{z}_3+z_3\bar{z}_1\right|^2 = \alpha+\beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $\alpha^2 + \beta^2$ is:

(1) 31 (2) 29

(3) 41 (4) 24

**Answer (2)**

**Sol.** $|z| = 1$

$\arg(z_1)=-\frac{\pi}{4}$, $\arg(z_2)=0$, $\arg(z_3)=\frac{\pi}{4}$

$z_1 = |1|e^{-\frac{\pi}{4}} = \frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}$

$z_2 = 1 + 0i$

$z_3 = \frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}$

$z_1\bar{z}_2 = \frac{1-i}{\sqrt{2}}$

$z_2\bar{z}_3 = \frac{1-i}{\sqrt{2}}$

$z_3\bar{z}_1 = \frac{(1+i)^2}{2}$

$z_1\bar{z}_2+z_2\bar{z}_3+z_3\bar{z}_1 = \sqrt{2}+i\left(1-\sqrt{2}\right)$

$\left|z_1\bar{z}_2+z_2\bar{z}_3+z_3\bar{z}_1\right|^2 = 5-2\sqrt{2}$

$\alpha = 5, \beta = -2$

$\alpha^2 + \beta^2 = 29$

12. Let $f : R \to R$ be a twice differentiable function such that $f(x + y) = f(x) f(y)$ for all $x, y \in R$. If $f'(0) = 4a$ and $f$ satisfies $f''(x) - 3af'(x) - f(x) = 0$, $a > 0$, then the area of the region $R = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is:

(1) $e^2 - 1$ (2) $e^4 - 1$

(3) $e^2 + 1$ (4) $e^4 + 1$

**Answer (1)**

**Sol.** $f(x + y) = f(x) f(y)$

$\Rightarrow$ Take $f(x) = k^x$

$k^x \log(k) = f'(x)$

$\log k = 4a \Rightarrow k = e^{4a}$

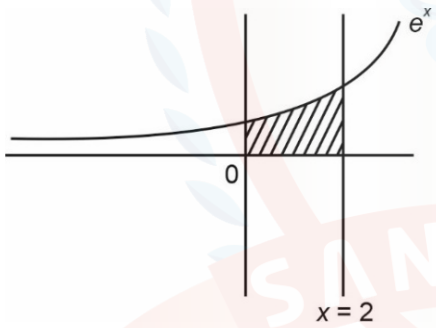
$f(x) = e^{4ax}$

$f''(x) - 3af'(x) - f(x) = 0$

$16a^2e^{4ax} - 12a^2e^{4ax} - e^{4ax} = 0$

$\Rightarrow \quad a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

$f(x) = e^{2x}$




$\int_0^2 (e^x)dx = \boxed{e^2 - 1}$

13. Let $x = x(y)$ be the solution of the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is

(1) $\frac{3}{2} + e$ (2) $3 - e$

(3) $3 + e$ (4) $\frac{1}{2} + e$

**Answer (2)**

**Sol.** $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$

$y^2dx = \left(\frac{1}{y} - x\right)dy$

$\Rightarrow y^2 \frac{dx}{dy} = \frac{1}{y} - x$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2}dy} = e^{\frac{-1}{y}}$

$\therefore$ Solution is

$xe^{\frac{-1}{y}} = \int e^{-\frac{1}{y}} \times \frac{1}{y^3}dy + C$

Let $\frac{-1}{y} = t$

$\Rightarrow \frac{1}{y^2}dy = dt$

$\Rightarrow xe^{-\frac{1}{y}} = -\int e^t t dt + C$

$\Rightarrow xe^{-\frac{1}{y}} = -e^t(t - 1) + C$

$\Rightarrow xe^{-\frac{1}{y}} = -e^{\frac{-1}{y}}\left(\frac{-1}{y} - 1\right) + C$

$x(1) = 1$

$\Rightarrow e^{-1} = -e^{-1}(-2) + C$

$\Rightarrow C = -e^{-1}$

$\Rightarrow x = \frac{1}{y} + 1 - e^{-1+\frac{1}{y}}$

$x\left(\frac{1}{2}\right) = 3 - e$

14. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where $gcd(m, n) = 1$, then $m + n$ is equal to

(1) 13 (2) 11

(3) 14 (4) 4

**Answer (3)**

**Sol.** Bag contains 4 white and 6 black balls

$A$ : first ball selected is black

$B$ : Second ball is also black

$$P\left(\frac{A}{B}\right)=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}=\frac{30}{24+30}$$

$$=\frac{30}{54}=\frac{5}{9}$$

$m + n = 5 + 9 = 14$

15. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points $P$, $Q$ and $R$. If the circle $C$ with centre at $(-1, -1)$ passes through the points $P$, $Q$ and $R$, then the area of $\Delta PQR$ is

(1) 5 (2) 4

(3) 7 (4) 6

**Answer (4)**

**Sol.** $y = x^2 + px - 3$ is a parabola

at $x = 0 \Rightarrow y = -3$

Parabola cuts the $y$-axis at $(0, -3)$

Equation of circle will be

$(x + 1)^2 + (y + 1)^2 = (0 + 1)^2 + (-3 + 1)^2$

$\Rightarrow (x + 1)^2 + (y + 1)^2 = 5$

$\Rightarrow x^2 + y^2 + 2x + 2y - 3 = 0$

Now this circle cuts $x$-axis at $y = 0$

$x^2 + 2x - 3 = 0$

$\Rightarrow (x + 3)(x - 1) = 0$

$\Rightarrow x = -3, 1$

$(-3, 0)$ and $(1, 0)$

Points $P$, $Q$ and $R$ are $(0, -3)$ $(-3, 0)$ and $(1, 0)$

$$\text{Area of } (\Delta PQR) = \frac{1}{2}\begin{vmatrix} 0 & -3 & 1 \\ -3 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\times 12 = 6$$

$\Rightarrow$ Option (4) is correct.

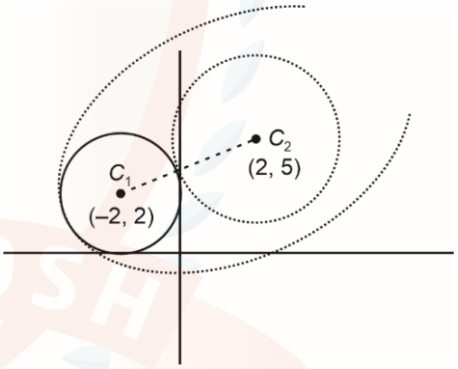
16. A circle $C$ of radius 2 lies in the second quadrant and touches both the coordinate axes. Let $r$ be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle $C$ at exactly two points. If the set of all possible values of $r$ is the interval $(\alpha, \beta)$, then $3\beta - 2\alpha$ is equal to:

(1) 12 (2) 14

(3) 15 (4) 10

**Answer (3)**

**Sol.**




$$C_1C_2 = \sqrt{(2+2)^2 + (5-2)^2}$$

$$= \sqrt{16+9}$$

$= 5$

$r + 2 > 5$

$r > 3$

$r < 5 + 2$

$r < 7$

$\therefore \alpha = 3, \beta = 7$

$3\beta - 2\alpha = 3(7) - 2(3) = 21 - 6 = 15$

17. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{1}{T_r}\right)$ is equal to

(1) 0 (2) 1

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

**Answer (4)**

**Sol.** $T_n = S_n - S_{n-1}$

$$= \frac{(2n-1)(2n+1)(2n+3)(2n+5) - (2n-3)(2n-1)(2n+1)(2n+3)}{64}$$

$$T_n = \frac{(2n-1)(2n+1)(2n+3)}{8}$$

$$\frac{1}{T_n} = 2\left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n-1)(2n+3)}\right)$$

$$S_n = 2\left(\frac{1}{1\times 3} - \frac{1}{(2n-1)(2n+3)}\right)$$

$$\lim_{n\to\infty} S_n = \frac{2}{3}$$

18. Let $A = \{1, 2, 3, \ldots, 10\}$ and $B = \left\{\frac{m}{n} : m, n \in A, m < n \text{ and } gcd(m, n)=1\right\}$. Then $n(B)$ is equal to

(1) 36 (2) 31

(3) 29 (4) 37

**Answer (2)**

**Sol.** Number of elements in set $B$, corresponding to

$m = 1$ are 9

$m = 2$ are $n = 3, 5, 7, 9 = 4$

$m = 3$ are $n = 4, 5, 7, 8, 10 = 5$

$m = 4$ are $n = 5, 7, 9 = 3$

$m = 5$ are $n = 6, 7, 8, 9 = 4$

$m = 6$ are $n = 7 = 1$

$m = 7$ are $n = 8, 9, 10 = 3$

$m = 8$ are $n = 9 = 1$

$m = 9$ are $n = 10 = 1$

Total number = 9 + 4 + 5 + 3 + 4 + 1 + 3 + 1 + 1 = 31

19. The product of all solutions of the equation $e^{5(\log_e x)^2+3} = x^8$, $x > 0$, is

(1) $e^{\frac{6}{5}}$ (2) $e$

(3) $e^{\frac{8}{5}}$ (4) $e^2$

**Answer (3)**

**Sol.** $e^{5(\log_e x)^2+3} = e^{\log x^8}$

$\Rightarrow 5(\log_e x)^2 + 3 = \log x^8 = 8\log_e x$

Put $\log_e x = t$, $5t(t-1) - 3(t-1) = 0$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$\Rightarrow 5t^2 - 5t - 3t + 3 = 0$

$(5t-3)(t-1) = 0$

$t = 1 \Rightarrow \log_e x = 1 \Rightarrow x = e$

$t = \frac{3}{5} \Rightarrow \log_e x = \frac{3}{5} \Rightarrow x = e^{\frac{3}{5}}$

Product of all roots = $e^{1+\frac{3}{5}} = e^{\frac{8}{5}}$

Option = 3

20. Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$, $I_1 = \int_0^{\frac{\pi}{4}} f(x)\,dx$ and $I_2 = \int_0^{\frac{\pi}{4}} x\, f(x)\,dx$. Then $7I_1 + 12I_2$ is equal to

(1) $\pi$ (2) $2\pi$

(3) 1 (4) 2

**Answer (3)**

**Sol.** $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$

$= 7\tan^6 x(1 + \tan^2 x) - 3\tan^2 x(1 + \tan^2 x)$

$= (7\tan^6 x - 3\tan^2 x)(1 + \tan^2 x)$

$= (7\tan^6 x - 3\tan^2 x)\sec^2 x$

$$I_1 = \int_0^{\frac{\pi}{4}} f(x)\,dx = \int_0^{\frac{\pi}{4}} (7\tan^6 x - 3\tan^2 x)\sec^2 x\,dx$$

$$= \left(\frac{7\tan^7 x}{7} - \frac{3\tan^3 x}{3}\right)_0^{\frac{\pi}{4}} = 1 - 1 = 0$$

$$I_2 = \int_0^{\frac{\pi}{4}} xf(x)\,dx = \int_0^{\frac{\pi}{4}} x(7\tan^6 x - 3\tan^2 x)\sec^2 x\,dx$$

$$= x(\tan^7 x - \tan^3 x)\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 1.(\tan^7 x - \tan^3 x)\,dx$$

$$= 0 - \int_0^{\frac{\pi}{4}} \tan^3 x(\tan^2 x - 1)(\tan^2 x + 1)\,dx$$

$$= \int_0^{\frac{\pi}{4}} (\tan^3 x - \tan^5 x)\sec^2 x\,dx = \frac{\tan^4 x}{4} - \frac{\tan^6 x}{6}\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{12}$$

Hence $7I_1 + 12I_2 = 1$

## SECTION - B

**Numerical Value Type Questions:** This section contains 5Numerical based questions. The answer to each question should be rounded off to the nearest integer.

21. Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}, \lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____.

**Answer (16)**

**Sol.** $\vec{c} = \left(\frac{\vec{b}.\vec{a}}{|\vec{b}|}\right)\frac{\vec{a}}{|\vec{a}|}$

$$= \left(\frac{\lambda + 8}{9}\right)\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

$$|\vec{a} + \vec{c}| = 7$$

$$\Rightarrow \left|\left(\frac{\lambda + 8}{9} + 1\right)\hat{i} + \left(\frac{2(\lambda + 8)}{9} + 2\right)\hat{j} + \left(\frac{2(\lambda + 8)}{9} + 2\right)\hat{k}\right| = 7$$

$$\left(\frac{\lambda + 8}{9} + 1\right)^2 + \left(\frac{2(\lambda + 8)}{9} + 2\right)^2 + \left(\frac{2(\lambda + 8)}{9} + 2\right)^2 = 49$$

$$\Rightarrow \lambda = 4 \Rightarrow \vec{c} = \frac{4}{3}\hat{i} + \frac{8}{3}\hat{j} + \frac{8}{3}\hat{k}$$

Area of parallelogram = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4 \end{vmatrix}$ = 16

22. If $\sum_{r=0}^{5} \frac{{}^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, $gcd(m, n) = 1$, then $m - n$ is equal to _____.

**Answer (2035)**

**Sol.** $(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \cdots + {}^{11}C_{11} x^{11}$

$$\int_0^1 (1+x)^{11}dx = \int_0^1 \left({}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \cdots + {}^{11}C_{11} x^{11}\right)dx$$

$$\frac{(1-x)^{12}}{12}\Big|_0^1 = {}^{11}C_{0x} + \frac{{}^{11}C_1 x^2}{2} + \frac{{}^{11}C_2 x^3}{3} + \cdots + \frac{{}^{11}C_{11} x^{12}}{12}\Big|_0^1$$

$$\boxed{\frac{2^{12} - 1}{12} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_{11}}{12}} \quad \ldots(1)$$

Now,

$$\int_{-1}^0 (1+x)^{11}dx = \int_{-1}^0 \left({}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \cdots + {}^{11}C_{11} x^{11}\right)dx$$

$$\frac{(1+x)^{12}}{12}\Big|_{-1}^0 = {}^{11}C_0 x + \frac{{}^{11}C_1 x^2}{2} + \frac{{}^{11}C_2 x^3}{3} + \cdots + \frac{{}^{11}C_{11} x^{12}}{12}\Big|_{-1}^0$$

$$\boxed{\frac{1}{12} = C_0 - \frac{C_1}{2} + \frac{C_2}{3}\cdots} \quad \ldots(2)$$

(1) – (2)

$$= \frac{2^{12} - 2}{12} = 2\left[\frac{C_1}{2} + \frac{C_3}{4} + \cdots\right]$$

$$\Rightarrow \sum_{r=0}^{5} \frac{C_{2r+1}}{2r+2} = \frac{2^{11} - 1}{12} = \frac{2047}{12} = \frac{m}{n}$$

= 2047 –12 = 2035

23. Let the function,

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\geq 1\end{cases}$$

be differentiable for all $x \in R$, where $a>1$, $b \in \boldsymbol{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha+\beta\sqrt{3}$, $\alpha, \beta \in Z$, then the value of $\alpha + \beta$ is _____.

**Answer (34)**

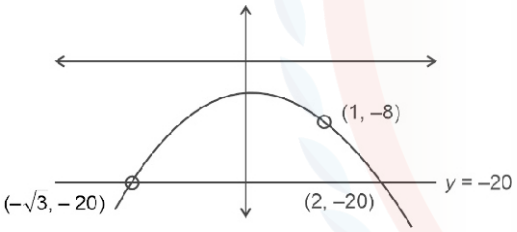
**Sol.** $f(x)$ is continuous and differentiable

at $x = 1$, LHL = RHL, LHD = RHD

$-3a - 2 = a^2 + b$, $-6a = b$

$a = 2$; $b = -12$

$$f(x)=\begin{cases}-6x^2-2, & x<1\\ 4-12x, & x\geq 1\end{cases}$$




$$\text{Area} = \int_{-\sqrt{3}}^{1}(-6x^2-2+20)\,dx+\int_{1}^{2}(4-12x+20)\,dx]$$

$$=16+12\sqrt{3}+6=22+12\sqrt{3}$$

$\therefore \quad \alpha + \beta = 34$

24. Let $L_1:\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $L_2:\frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}, \alpha\in R$, be two lines, which intersect at the point $B$. If $P$ is the foot of perpendicular from the point $A(1, 1, -1)$ and $L_2$, then the value of $26\,\alpha(PB)^2$ is

**Answer (216)**

**Sol.** Point $B$

$$(3\lambda+1,-\lambda+1,-1)\equiv(2\mu+2,\ 0,\ \alpha\mu-4)$$

$3\lambda + 1 = 2\mu + 2$

$-\lambda + 1 = 0$

$-1 = \alpha\,\mu - 4$

$\lambda = 1$, $\mu = 1$, $\alpha = 3$

$B\ (4, 0, -1)$

Let Point '$P$' is $(2\delta + 2, 0, 3\delta - 4)$

Dr's of AP $< 2\delta + 1, -1, 3\delta - 3 >$

$$AP \perp L_2 \Rightarrow \delta=\frac{7}{13}$$

$$P\left(\frac{40}{13},0,\frac{-31}{13}\right)$$

$$\therefore\ 26\alpha(PB)^2=26\times 3\times\left(\frac{144}{169}+\frac{324}{169}\right)$$

$= 216$

25. Let $A$ be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\text{adj}\,(-6\text{adj}\,(3A))) = 2^{m+n}.3^{mn}$, $m > n$. Then $4m + 2n$ is equal to _____.

**Answer (34)**

**Sol.** As $A\,\text{adj}A = |A|I$, $\det(\lambda A) = \lambda^n \det A$

$\det(3\text{adj}(-6\text{adj}(3A))) = 3^3\det(\text{adj}(-6\text{adj}(3A)))$

$= 3^3(-6\text{adj}(3A))^2$

$= 3^3(-6)^6|3A|^4$

$= 3^9 2^6.3^{12}.(-2)^4$

$= 3^{21}.2^{10}$

Now comparing with given condition

$2^{m+n}\,3^{mn} = 2^{10}.3^{21}$

$m + n = 10$, $mn = 21$

$\Rightarrow \quad m = 7$, $n = 3$ $(m>n)$

$\therefore \quad 4m + 2n = 28 + 6 = 34$

# PHYSICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

26. An electron is made to enter symmetrically between two parallel and equally but oppositely charged metal plates, each of 10 cm length. The electron emerge out of the electric field region with a horizontal component of velocity $10^6$ m/s. If the magnitude of the electric field between the plates is 9.1 V/cm, then the vertical component of velocity of electron is

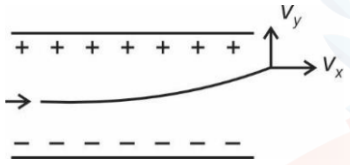
(mass of electron = $9.1 \times 10^{-31}$ kg and charge of electron = $1.6 \times 10^{-19}$ C)

(1) 0 (2) $16 \times 10^6$ m/s

(3) $16 \times 10^4$ m/s (4) $1 \times 10^6$ m/s

**Answer (2)**

**Sol.**




Horizontal component of velocity will remain same.

$$t = \frac{l}{v_x} = \frac{0.1}{10^6}$$

$$v_y = a_y t = \frac{eE}{m} t$$

$$= \frac{1.6 \times 10^{-19} \times 9.1 \times 10^2}{9.1 \times 10^{-31}} \times \frac{0.1}{10^6}$$

$$= 1.6 \times 10^7$$

27. An amount of ice of mass $10^{-3}$ kg and temperature –10°C is transformed to vapour of temperature 110°C by applying heat. The total amount of work required for this conversion is,

(Take, specific heat of ice = 2100 $Jkg^{-1}K^{-1}$, specific heat of water 4180 $Jkg^{-1}K^{-1}$, specific heat of steam = 1920 $Jkg^{-1}K^{-1}$, Latent heat of ice = $3.35 \times 10^5$ $Jkg^{-1}$ and Latent heat of steam = $2.25 \times 10^6$ $Jkg^{-1}$)

(1) 3022 J

(2) 3043 J

(3) 3024 J

(4) 3003 J

**Answer (2)**

**Sol.** ice at –10°C ⟶ ice at 0°C ⟶ water at 0°C ↓

vapour at 110°C ⟵ vapour at 100°C ⟵ water at 100°

$$Q = 10^{-3}\left[2100 \times 10 + 3.35 \times 10^5 + 100 \times 4180 + 2.25 \times 10^6 + 10 \times 1920\right]$$

= 3043.2 J

28. If $B$ is magnetic field and $\mu_0$ is permeability of free space, then the dimensions of $(B/\mu_0)$ is

(1) $ML^2T^{-2}A^{-1}$

(2) $L^{-1}A$

(3) $LT^{-2}A^{-1}$

(4) $MT^{-2}A^{-1}$

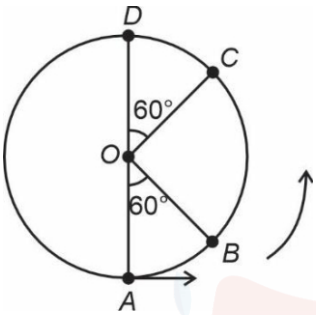
**Answer (2)**

**Sol.** For a current carrying loop at centre

$$B = \frac{\mu_0 i}{2R}$$

$$\therefore \quad \frac{B}{\mu_0} = \frac{i}{R} \quad [AL^{-1}]$$

29. A bob of mass $m$ is suspended at a point $O$ by a light string of length $l$ and left to perform vertical motion (circular) as shown in figure. Initially, by applying horizontal velocity $v_0$ at the point '$A$', the string becomes slack when, the bob reaches at the point '$D$'. The ratio of the kinetic energy of the bob at the point $B$ and $C$ is _____.




(1) 1

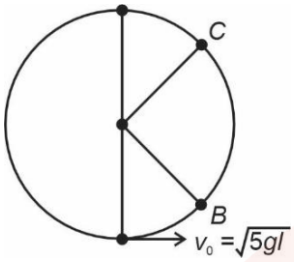
(2) 3

(3) 4

(4) 2

**Answer (4)**

**Sol.**



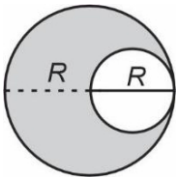

$$v_B = \sqrt{v_0^2 - 2g\left(\frac{l}{2}\right)}$$

$$v_B = \sqrt{4gl}$$

$$v_C = \sqrt{v_0^2 - 3gl} \quad \sqrt{2gl}$$

$$\frac{K_B}{K_C} = \frac{v_B^2}{v_C^2} \quad 2$$

30. A uniform circular disc of radius '$R$' and mass '$M$' is rotating about an axis perpendicular to its plane and passing through its centre. A small circular part of radius $R/2$ is removed from the original disc as shown in the figure. Find the moment of inertia of the remaining part of the original disc about the axis as given above.




(1) $\frac{7}{32}MR^2$ (2) $\frac{17}{32}MR^2$

(3) $\frac{13}{32}MR^2$ (4) $\frac{9}{32}MR^2$

**Answer (3)**

**Sol.**

$$I_{\text{complete}} = \frac{MR^2}{2}$$

$$I_{\text{removed}} = \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 \frac{1}{2}^2 \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)$$

$$= \frac{3MR^2}{32}$$

$$I = \frac{MR^2}{2} - \frac{3MMR}{32} \quad \frac{13}{32}MR^2$$

31. A parallel-plate capacitor of capacitance 40μF is connected to a 100 V power supply. Now the intermediate space between the plates is filled with a dielectric material of dielectric constant K = 2. Due to the introduction of dielectric material, the extra charge and the change in the electrostatic energy in the capacitor, respectively, are

(1) 2 mC and 0.4 J (2) 8 mC and 2.0 J

(3) 4 mC and 0.2 J (4) 2 mC and 0.2 J

**Answer (3)**

**Sol.** $C = 40\ \mu F$

$C' = kC = 80\ \mu F$

$V = 100\ V$

$Q = CV = 4\ mC$

$Q' = C'V = 8\ mC$

$\therefore$ Extra charge = 4 mC

$$U_1 = \frac{1}{2}CV^2, U^2 = -C'V$$

$$\Delta U = \frac{1}{2}(C' - C)V^2$$

$$= \frac{1}{2} \times 40 \times 10^4 \times 10^{-6}$$

$$= 0.2\ J$$

32. The work functions of cesium (Cs) and lithium (Li) metals are 1.9 eV and 2.5 eV, respectively. If we incident a light of wavelength 550 nm on these two metal surfaces, then photo-electric effect is possible for the case of

(1) Neither Cs nor Li (2) Both Cs and Li

(3) Cs only (4) Li only

**Answer (3)**

**Sol.** $E$(Energy of incident photon) $= \frac{1240}{550} = 2.25$ eV

$E > \phi_{Cs}$, $< \phi_{Li}$

$\therefore$ Only cesium will show photoelectric effect.

33. An electron in the ground state of the hydrogen atom has the orbital radius of $5.3 \times 10^{-11}$ m while that for the electron in third excited state is $8.48 \times 10^{-10}$ m. The ratio of the de Broglie wavelengths of electron in the ground state to that in the excited state is

(1) 4 (2) 16

(3) 3 (4) 9

**Answer (*)**

**Sol.** According to de-Broglie hypothesis

$$2\pi r_n = n\lambda$$

$$\frac{2\pi r_1}{2\pi r_4} = \frac{\lambda_1}{4\lambda_4}$$

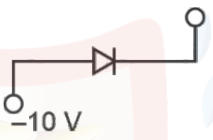
$$\frac{\lambda_1}{\lambda_4} = 4\left(\frac{r}{r}\right)$$

$$r \propto n^2$$

$$\therefore \quad \frac{\lambda_1}{\lambda_4} = \frac{1}{4}$$

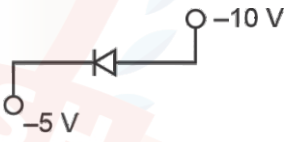
*** None of the option is matching.**

34. Which of the following circuits represents a forward biased diode?

(A) 


(B) 


(C) 


(D) 


(E) 


Choose the **correct** answer from the options given below:

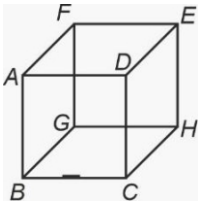
(1) (B), (D) and (E) only (2) (A) and (D) only

(3) (C) and (E) only (4) (B), (C) and (E) only

**Answer (4)**

**Sol.** For forward bias potential of *p* side should be higher than *n* side.

35. A line charge of length $\frac{'a'}{2}$ is kept at the center of an edge *BC* of a cube ABCDEFGH having edge length '*a*' as shown in the figure. If the density of line charge is $\lambda C$ per unit length, then the total electric flux through all the faces of the cube will be _____. (Take, $\varepsilon_0$ as the free space permittivity)




(1) $\frac{\lambda a}{16\varepsilon_0}$ (2) $\frac{\lambda a}{2\varepsilon_0}$

(3) $\frac{\lambda a}{4\varepsilon_0}$ (4) $\frac{\lambda a}{8\varepsilon_0}$

**Answer (4)**

**Sol.** Charge of the line charge = $\frac{a\lambda}{2}$

Portion of wire inside cube = $\frac{1}{4}$

$\therefore \quad q_{en} = \frac{1}{4}\left(\frac{a\lambda}{2}\right) = \frac{a\lambda}{8}$

$$\phi = \frac{q_{en}}{\varepsilon_0} = \frac{a\lambda}{\varepsilon_0 8}$$

36. Given below are two statements :

**Statement-I :** The equivalent emf of two nonideal batteries connected in parallel is smaller than either of the two emfs.

**Statement-II :** The equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement-I is false but Statement-II is true

(2) Statement-I is true but Statement-II is false

(3) Both Statement-I and Statement-II are true

(4) Both Statement-I and Statement-II are false

**Answer (1)**

**Sol.** In parallel connections

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

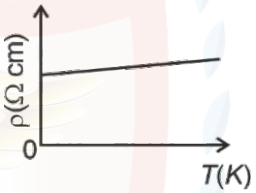
If $E_1 = E_2$ and $r_1 = r_2$, $E_{eq} = E_1 = E_2$

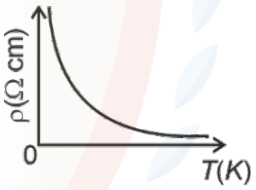
$\therefore$ Statement I is false.

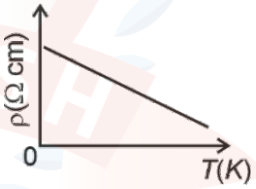
$r_{eq}$ is less then both $r_1$ and $r_2$

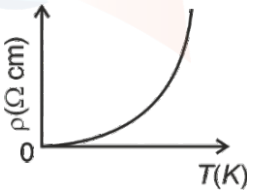
$\therefore$ Statement II is true

37. Which of the following resistivity ($\rho$) v/s temperature ($T$) curves is most suitable to be used in wire bound standard resistors?

(1)



(2)



(3)



(4)



**Answer (1)**

**Sol.** For bound standard resistors the resistivity will remain nearly constant.

38. A closed organ and an open organ tube are filled by two different gases having same bulk modulus but different densities $\rho_1$ and $\rho_2$, respectively. The frequency of 9th harmonic of closed tube is identical with 4th harmonic of open tube. If the length of the closed tube is 10 cm and the density ratio of the gases is $\rho_1 : \rho_2 = 1 : 16$, then the length of the open tube is :

(1) $\frac{15}{7}$ cm (2) $\frac{20}{9}$ cm

(3) $\frac{15}{9}$ cm (4) $\frac{20}{7}$ cm

**Answer (2)**

**Sol.** $9\left(\frac{v_c}{4l_c}\right) = \frac{v_o}{2l_o}$

$v = \sqrt{\frac{B}{\rho}}$

$\therefore \quad \frac{v_c}{v_o} = \sqrt{\frac{\rho_2}{\rho_1}} = 4$

$l_o = \frac{8}{9}\left(\frac{v_o}{v_c}\right)$

$= \frac{8}{9} l_c \times \frac{1}{4} = \frac{2}{9}$

$l_o = \frac{20}{9}$ cm

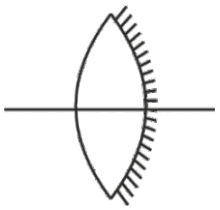
39. Given is a thin convex lens of glass (refractive index $\mu$) and each side having radius of curvature $R$. One side is polished for complete reflection. At what distance from the lens, an object be placed on the optic axis so that the image gets formed on the object itself?

(1) $R/\mu$ (2) $\mu R$

(3) $R/(2\mu - 1)$ (4) $R/(2\mu - 3)$

**Answer (3)**

**Sol.**



$-\frac{1}{f_{eq}} = \frac{2}{f_l} - \frac{1}{f_m}$

$= 2(\mu - 1)\frac{2}{R} + \frac{2}{R}$

$-\frac{1}{f_{eq}} = \frac{2(2\mu - 1)}{R}$

$f_{eq} = -\frac{R}{2(2\mu - 1)}$

For concave mirror, object should be at $2f$ for the image to be at same point

Distance $= \frac{R}{(2\mu - 1)}$

40. Two spherical bodies of same materials having radii 0.2 m and 0.8 m are placed in same atmosphere. The temperature of the smaller body is 800 K and temperature of the bigger body is 400 K. If the energy radiated from the smaller body is $E$, the energy radiated from the bigger body is (assume, effect of the surrounding temperature to be negligible),

(1) $E$ (2) 64 $E$

(3) 256 $E$ (4) 16 $E$

**Answer (1)**

**Sol.** $P = \sigma \varepsilon A T^4$

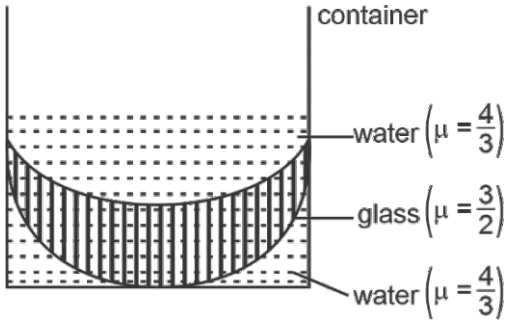
$\frac{P_1}{P_2} = \frac{R_1^2 T_1^4}{R_2^2 T_2^4}$

$\frac{P_1}{P_2} = \frac{1}{16} \times 2^4$

$P_1 = P_2$

$\therefore$ Energy will be same

41. In the diagram given below, there are three lenses formed. Considering negligible thickness of each of them as compared to $|R_1|$ and $|R_2|$, *i.e.*, the radii of curvature for upper and lower surface of the glass lens, the power of the combination is




(1) $\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

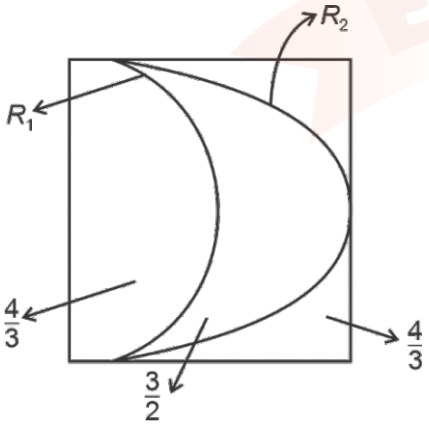
(2) $-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

(3) $-\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$

(4) $\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$

**Answer (2)**

**Sol.**



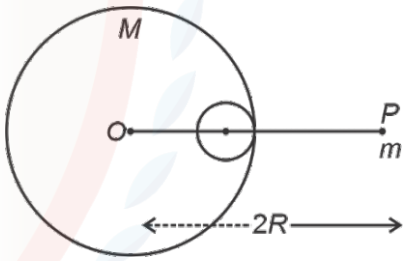

$$P_{eq} = P_1 + P_2 + P_3$$

$$= \left(\frac{4}{3} - 1\right)\left(\frac{1}{R_1} - 1\right) + \frac{1}{2}\left(-\frac{1}{R_1} + \frac{1}{R_2}\right) + \left(\frac{4}{3} - \right)\left(-\frac{1}{R_2}\right)$$

$$= \frac{1}{3}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P_{eq} = -\frac{1}{6}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

42. A small point of mass $m$ is placed at a distance $2R$ from the centre '$O$' of a big uniform solid sphere of mass $M$ and radius $R$. The gravitational force on '$m$' due to $M$ is $F_1$. A spherical part of radius $\frac{R}{3}$ is removed from the big sphere as shown in the figure and the gravitational force on $m$ due to remaining part of $M$ is found to be $F_2$. The value of ratio $F_1 : F_2$ is




(1) 12 : 9

(2) 11 : 10

(3) 12 : 11

(4) 16 : 9

**Answer (3)**

**Sol.** $F_1 = \frac{GMm}{4R^2}$

$$F_2 = \frac{GMm}{4R^2} \quad \left(\frac{M}{27}\right) \frac{m}{\left(\frac{4R}{3}\right)}$$

$$= \frac{GMm}{R^2}\left(\frac{1}{4}\right)\frac{1}{48}$$

$$= \frac{11}{48}\,\frac{GMm}{R^2}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{12}{11}$$

43. Given below are two statements :

**Statement-I:** In a vernier callipers, one vernier scale division is always smaller than one main scale division.

**Statement-II:** The vernier constant is given by one main scale division multiplied by the number of vernier scale divisions.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement-I and Statement-II are true

(2) Both Statement-I and Statement-II are false

(3) Statement-I is false but Statement-II is true

(4) Statement-I is true but Statement-II is false

**Answer (2)**

**Sol.** 1 vernier scale division is not always smaller than 1 main scale divisions statement-I is not correct.

Vernier constant = 1 main scale division – 1 vernier scale division

Statement-II is not true

44. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

**Assertion-(A):** If Young's double slit experiment is performed in an optically denser medium than air, then the consecutive fringes come closer.

**Reason-(R):** The speed of light reduces in an optically denser medium than air while its frequency does not change.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(4) **(A)** is false but **(R)** is true

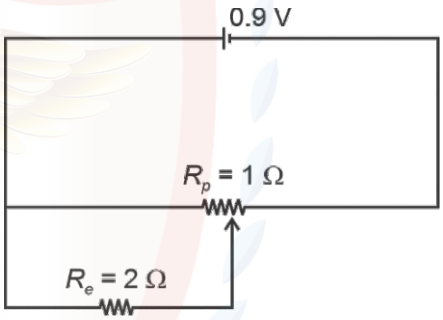
**Answer (1)**

**Sol.** $\beta = \frac{\lambda D}{d}$

In denser medium $\lambda' = \frac{\lambda}{\mu}$

$\therefore$ $\beta$ decreases.

45.




Sliding contact of a potentiometer is in the middle of the potentiometer wire having resistance $R_p$ = 1 Ω as shown in the figure. An external resistance of $R_e$ = 2 Ω as connected via the sliding contact. The electric current in the circuit is :

(1) 1.35 A

(2) 1.0 A

(3) 0.3 A

(4) 0.9 A

**Answer (2)**

**Sol.** $R_{eq} = (0.5\ \Omega \,||\, 2\ \Omega) + 0.5\ \Omega$

$$= \frac{2}{5} + \frac{1}{2} = \frac{9}{10}\ \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{0.9}{0.9} = 1\ \text{A}$$

## SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. The position vectors of two 1 kg particles, ($A$) and ($B$), are given by $r_A = \left(\alpha_1 t^2\,\hat{i} + \alpha_2 t\,\hat{j} + \alpha_3 t\,\hat{k}\right)$ m and $r_B = \left(\beta_1 t\,\hat{i} + \beta_2 t^2\,\hat{j} + \beta_3 t\,\hat{k}\right)$ m, respectively; ($\alpha_1$ = 1 m/s$^2$, $\alpha_2 = 3n$ m/s$^2$, $\alpha_3$ = 2 m/s, $\beta_1$ = 2 m/s, $\beta_2$ = –1 m/s$^2$, $\beta_3 = 4p$ m/s), where $t$ is time, $n$ and $p$ are constants. At $t$ = 1 s, $|V_A| = |V_B|$ and velocities $V_A$ and $V_B$ of the particles are orthogonal to each other. At $t$ = 1 s, the magnitude of angular momentum of particle ($A$) with respect to the position of particle ($B$) is $\sqrt{L}$ kgm$^2$s$^{-1}$. The value of $L$ is _____.

**Answer (90)**

**Sol.** At $t$ = 1

$$r_{AB} = -1\hat{i} + (3n+1)\hat{j} + (2-4p)\hat{k}$$

At $t$ = 1

$$v_A = 2\hat{i} + 3n\,\hat{j} + 2\hat{k}$$

$$v_B = 2\hat{i} - 2\hat{j} + 4p\,\hat{k}$$

$v_A = v_B$ ⇒ 0 = , $4 - 6n + 8p = 0$

$|v_A| = |v_B|$ ⇒ $(3n)^2 + 4 = 4 + 16p^2$

$3n = -4p$

$4 + 16p = 0$

$p = -\frac{1}{4}, \frac{1}{3} = -$

$$r_{AB} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$v_A = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore\ L = m\left|r_{AB} \times v_A\right| = 90$$

47. The driver sitting inside a parked car is watching vehicles approaching from behind with the help of his side view mirror, which is a convex mirror with radius of curvature $R$ = 2 m. Another car approaches him from behind with a uniform speed of 90 km/hr. When the car is at a distance of 24 m from him, the magnitude of the acceleration of the image of the car in the side view mirror is '$a$'. The value of 100$a$ is _____ m/s$^2$.

**Answer (8)**

**Sol.** $\frac{1}{V} + \frac{1}{U} = \frac{1}{f}$

$$\frac{1}{V} - \frac{1}{24} = 1$$

$$V = \frac{24}{25}$$

$$m = -\frac{V}{U} = \frac{1}{25}$$

$$|V_{\text{image}}| = m^2 V_{\text{object}} = \frac{1}{25}\ \text{m/s}$$

$$\frac{1}{V}+\frac{1}{U}=\frac{1}{f}$$

$$\frac{d^2V}{dt^2}=V^2\left[\frac{2}{V^3}\left(\frac{dV}{dt}\right)^2-\frac{2}{U^3}\left(\frac{dU}{dt}\right)\right]$$

$$a=-\frac{2}{25}$$

$$|100a|=8$$

48. A particle is projected at an angle of 30° from horizontal at a speed of 60 m/s. The height traversed by the particle in the first second is $h_0$ and height traversed in the last second, before it reaches the maximum height, is $h_1$. The ratio $h_0 : h_1$ is _____.

[Take, $g$ = 10 m/s$^2$]

**Answer (5)**

**Sol.** $U$ = 60 m/s

$\theta = 30°$

$$t_{H_{max}}=\frac{U\sin\theta}{g}=\frac{60}{10}\times\frac{1}{2}=3\text{s}$$

Height in 1st second = $30\times1-\frac{1}{2}\times10\times1$

= 25 m

Height in last second = 5 m

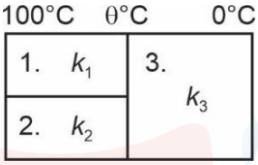
$$\frac{25}{5}=5$$

49. Two soap bubbles of radius 2 cm and 4 cm, respectively, are in contact with each other. The radius of curvature of the common surface, in cm, is _____.

**Answer (4)**

**Sol.** $R=\frac{R_2R_1}{R_2-R_1}=\frac{4\times2}{2}=4$ cm

50. Three conductors of same length having thermal conductivity $k_1$, $k_2$ and $k_3$ are connected as shown in figure.




Area of cross sections of 1st and 2nd conductor are same and for 3rd conductor it is double of the 1st conductor. The temperature are given in the figure. In steady state condition, the value of θ is _____ °C.

(Given : $k_1$ = 60 Js$^{-1}$m$^{-1}$K$^{-1}$, $k_2$ = 120 Js$^{-1}$m$^{-1}$K$^{-1}$, $k_3$ = 135 Js$^{-1}$m$^{-1}$K$^{-1}$)

**Answer (40)**

**Sol.** $R_1=\frac{2}{k_1A}, R_2=\frac{2}{k_2A}$ $R_3=\frac{}{k_3A}$

$$(100-\theta)\left(\frac{1}{R_1}\right)+\frac{1}{R_2}=\frac{(\theta-0)}{R_2}$$

$$(100-\theta)\frac{(k_1+k_2)A}{2}=\frac{\theta k_3A}{}$$

$$(100-\theta)(k_1+k_2)=2\theta k_3$$

$$\theta=\frac{100(k_1+k_2)}{k_1+k_2+2k_3}$$

$$=\frac{100\times180}{450}=40$$

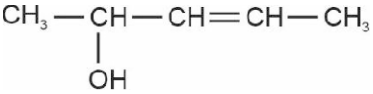
# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

51. How many different stereoisomers are possible for the given molecule?



(1) 2 (2) 4

(3) 3 (4) 1

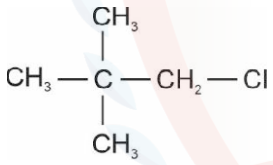
**Answer (2)**

**Sol.** n = 2

Number of stereoisomers = $(2)^2 = 4$

52. Given below are two statements:

**Statement I** : $CH_3 - O - CH_2 - Cl$ will undergo $S_N1$ reaction though it is a primary halide.

**Statement II** :  will not undergo $S_N2$ reaction very easily though it is a primary halide.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both **Statement I** and **Statement II** are incorrect

(2) Both **Statement I** and **Statement II** are correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) **Statement I** is incorrect but **Statement II** is correct

**Answer (2)**

**Sol.** Both statement-I and statement-II are correct

**Statement –** I :  is highly stable carbocation.

**Statement –** II : Due to hindrance, $S_N2$ will not take place easily.

53. A liquid when kept inside a thermally insulated closed vessel at 25ºC was mechanically stirred from outside. What will be the correct option for the following thermodynamic parameters?

(1) $\Delta U > 0, q = 0, w > 0$ (2) $\Delta U < 0, q = 0, w > 0$

(3) $\Delta U = 0, q < 0, w > 0$ (4) $\Delta U = 0, q = 0, w = 0$

**Answer (1)**

**Sol.** $q = 0$

$w > 0$

$\Delta U > 0$

54. In which of the following complexes the CFSE, $\Delta_0$ will be equal to zero?

(1) $K_3[Fe(SCN)_6]$ (2) $[Fe(en)_3]Cl_3$

(3) $[Fe(NH_3)_6]Br_2$ (4) $K_4[Fe(CN)_6]$

**Answer (1)**

**Sol.** $K_3[Fe(SCN)_6]$ will have CFSE equal to zero ($Fe^{3+}$ : $4s^0 3d^5 \left(t_{2g}^3 e_g^2\right)$

55. Which of the following electronegativity order is **incorrect**?

(1) Mg < Be < B < N (2) Al < Si < C < N

(3) Al < Mg < B < N (4) S < Cl < O < F

**Answer (3)**

**Sol.** Electronegativity of Al is higher than that of Mg

$\Rightarrow$ option (3) is correct

56. The **incorrect** statements regarding geometrical isomerism are

(A) Propene shows geometrical isomerism.

(B) Trans isomer has identical atoms/groups on the opposite sides of the double bond.

(C) Cis-but-2-ene has higher dipole moment than trans-but-2-ene.

(D) 2-methylbut-2-ene shows two geometrical isomers.

(E) Trans-isomer has lower melting point than cis isomer.

Choose the **correct** answer from the options given below.

(1) (A), (D) and (E) only (2) (A) and (E) only

(3) (C), (D) and (E) only (4) (B) and (C) only

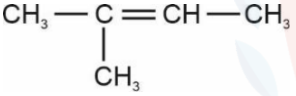
**Answer (1)**

**Sol.** Statement (A), (D) and (E) are incorrect statements.

(A) Propene does not show geometrical isomerism. (A) is incorrect.

(B) (B) is correct.

(C) Dipole moment : cis-but-2-ene > trans-but-2-ene

(D) 

2-methylbut-2-ene does not show geometrical isomerism. (D) is incorrect.

(E) Trans-isomer has higher melting point than cis-isomer. (E) is incorrect.

$\Rightarrow$ (A), (D) and (E) are incorrect.

57. From the magnetic behaviour of $[NiCl_4]^{2-}$ (paramagnetic) and $[Ni(CO)_4]$ (diamagnetic), choose the correct geometry and oxidation state.

(1) $[NiCl_4]^{2-}$ : $Ni^{II}$, square planar
$[Ni(CO)_4]$ : Ni(0), square planar

(2) $[NiCl_4]^{2-}$ : $Ni^{II}$, tetrahedral
$[Ni(CO)_4]$ : Ni(0), tetrahedral

(3) $[NiCl_4]^{2-}$ : Ni(0), tetrahedral
$[Ni(CO)_4]$ : Ni(0), square planar

(4) $[NiCl_4]^{2-}$ : $Ni^{II}$, tetrahedral
$[Ni(CO)_4]$ : $Ni^{II}$, square planar

**Answer (2)**

**Sol.** $[NiCl_4]^{2-}$ $\Rightarrow Ni^{2+} : 4s^0 3d^8$

$\Rightarrow$ No pairing

$\Rightarrow sp^3$

$\Rightarrow$ Tetrahedral

$[Ni(CO)_4]$ $\Rightarrow Ni^0 : 4s^0 3d^{10}$

$\Rightarrow sp^3$

$\Rightarrow$ Tetrahedral

58. Given below are two statements :

**Statement I :** One mole of propyne reacts with excess of sodium to liberate half a mole of $H_2$ gas.

**Statement II :** Four g of propyne reacts with $NaNH_2$ to liberate $NH_3$ gas which occupies 224 mL at STP.

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are incorrect

**Answer (3)**

**Sol.** $CH_3 - C \equiv CH + Na \rightarrow CH_3 - C \equiv C^-{}^{Na^+} + \frac{1}{2}H_2$

Statement-I is correct.

Moles of $C_3H_4$ = $\frac{4}{40}$ = 0.1 mole

$$\underset{0.1\,mole}{CH_3 - C \equiv CH} + NaNH_2 \rightarrow CH_3 - C \equiv C^-{}^{Na^+} + \underset{0.1\,mole}{NH_3}$$

Volume of $NH_3$ = (0.1)(22.4) = 2.24 L

Statement-II is incorrect.

59. Lanthanoid ions with $4f^7$ configuration are

(A) $Eu^{2+}$ (B) $Gd^{3+}$

(C) $Eu^{3+}$ (D) $Tb^{3+}$

(E) $Sm^{2+}$

Choose the correct answer from the options given below.

(1) (A) and (B) only

(2) (A) and (D) only

(3) (B) and (E) only

(4) (B) and (C) only

**Answer (1)**

**Sol.** $Eu^{2+}$ and $Gd^{3+}$ have $4f^7$ configuration.

60. Match List-I with List-II.

| List-I | List-II |
|---|---|
| (A) $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$ | (I) Ionisation enthalpy |
| (B) B < C < O < N | (II) Metallic character |
| (C) B < Al < Mg < K | (III) Electronegativity |
| (D) Si < P < S < Cl | (IV) Ionic radii |

Choose the correct **answer** from the options given below.

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(2) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

**Answer (4)**

**Sol.** (A) is order of ionic radii. A-(IV)

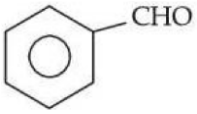
(B) is correct order of ionisation enthalpy. B-(I)

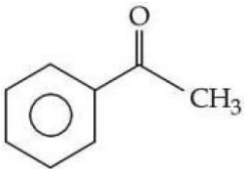
(C) is correct order of metallic character. C-(II)

(D) is correct order of electronegativity. D-(III)

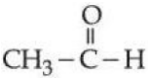
Option (4) is correct.

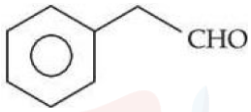
61. The compounds which give positive Fehling's test are:

(A) 

(B) 

(C) $HOCH_2 - CO - (CHOH)_3 - CH_2 - OH$

(D) 

(E) 

Choose the **correct** answer from the options given below:

(1) (C), (D) and (E) Only

(2) (A), (B) and (C) Only

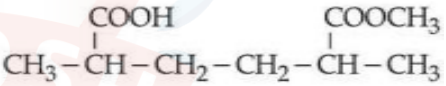
(3) (A), (D) and (E) Only

(4) (A), (C) and (D) Only

**Answer (1)**

**Sol.** Compound (C), (D) and (E) will give positive Fehling's test.

62. The IUPAC name of the following compound is:



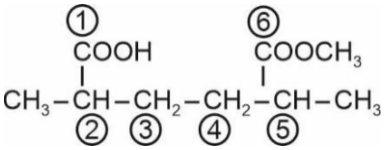
(1) Methyl-5-carboxy-2-methylhexanoate.

(2) 6-Methoxycarbonyl-2,5-dimethylhexanoic acid.

(3) Methyl-6-carboxy-2,5-dimethylhexanoate.

(4) 2-Carboxy-5-methoxycarbonylhexane.

**Answer (2)**

**Sol.** 

6-Methoxycarbonyl-2,5-dimethylhexanoic acid.

63. A solution of aluminium chloride is electrolysed for 30 minutes using a current of 2A. The amount of the aluminium deposited at the cathode is _____.

[Given: molar mass of aluminium and chlorine are 27 g $mol^{-1}$ and 35.5 g $mol^{-1}$ respectively. Faraday constant = 96500 C $mol^{-1}$]

(1) 0.336 g (2) 0.441 g

(3) 1.007 g (4) 1.660 g

**Answer (1)**

**Sol.** $Al^{3+} + 3e^- \longrightarrow Al$

$$\text{Moles of electron} = \frac{2 \times 30 \times 60}{96500}$$

$$= \frac{36}{965}$$

$$\text{Moles of Al} = \frac{36}{3 \times 965}$$

$$= \frac{12}{965}$$

$$\text{Mass of Al} = \frac{12}{965} \times 27$$

= 0.336 gm

64. Which of the following statement is not true for radioactive decay?

(1) Half life is ln 2 times of $\frac{1}{\text{rate constant}}$.

(2) Decay constant increases with increase in temperature.

(3) Amount of radioactive substance remained after three half lives is $\frac{1}{8}$th of original amount.

(4) Decay constant does not depend upon temperature.

**Answer (2)**

**Sol.** For radioactive decay, decay constant does not depend upon temperature because for radioactive decay activation energy is zero.

65. Radius of the first excited state of Helium ion is given as:

$a_0 \rightarrow$ radius of first stationary state of hydrogen atom.

(1) $r = \frac{a_0}{2}$ (2) $r = \frac{a_0}{4}$

(3) $r = 2a_0$ (4) $r = 4a_0$

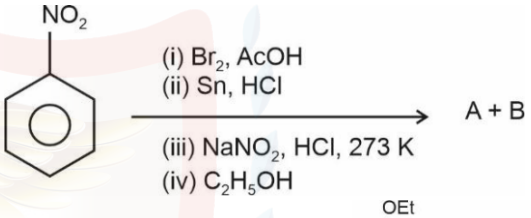
**Answer (3)**

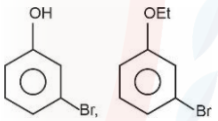
**Sol.** $r = a_0\left(\frac{n^2}{z}\right)$

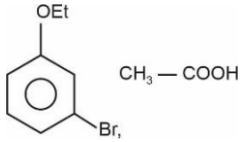
$$r = \frac{a_0(2)^2}{(2)}$$

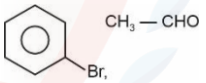
$r = 2a_0$

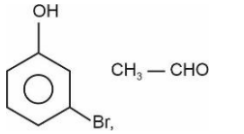
66. The products formed in the following reaction sequence are:



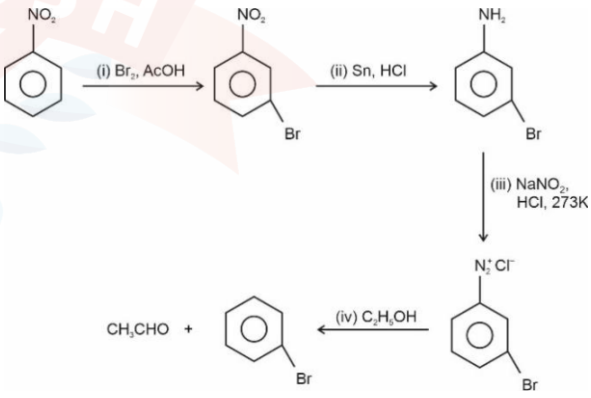

(1) 


(2) 


(3) 


(4) 


**Answer (3)**

**Sol.** 


Option (3) is correct

67. Which of the following electrolyte can be used to obtain $H_2S_2O_8$ by the process of electrolysis?
(1) Dilute solution of sulphuric acid
(2) Dilute solution of sodium sulphate
(3) Acidified dilute solution of sodium sulphate
(4) Concentrated solution of sulphuric acid

**Answer (4)**

**Sol.** $H_2S_2O_8$ is obtained by electrolysis of concentrated solution of sulphuric acid.

68. Arrange the following solutions in order of their increasing boiling points.
(i) $10^{-4}$ M NaCl (ii) $10^{-4}$ M Urea
(iii) $10^{-3}$ M NaCl (iv) $10^{-2}$ M NaCl
(1) (ii) < (i) ≡ (iii) < (iv) (2) (i) < (ii) < (iii) < (iv)
(3) (iv) < (iii) < (i) < (ii) (4) (ii) < (i) < (iii) < (iv)

**Answer (4)**

**Sol.** $\Delta T_b = i(K_b(m))$

$\Delta T_b$ order or $T_b$ order

(iv) > (iii) > (i) > (ii)

69. A vessel at 1000 K contains $CO_2$ with a pressure of 0.5 atm. Some of $CO_2$ is converted into CO on addition of graphite. If total pressure at equilibrium is 0.8 atm, then Kp is:
(1) 0.18 atm (2) 1.8 atm
(3) 0.3 atm (4) 3 atm

**Answer (2)**

**Sol.**

$$CO_{2(g)} + C(s) \rightleftharpoons 2CO_{(g)}$$

0.5 atm —

↓ ↓

(0.5 – x) (2x)

$(0.5 - x) + (2x) = 0.8$

$0.5 + x = 0.8$

$\boxed{x = 0.3}$

$K_p = \frac{(0.6)(0.6)}{(0.2)} = 1.8$ atm

70. Which of the following acids is a vitamin?
(1) Saccharic acid (2) Adipic acid
(3) Aspartic acid (4) Ascorbic acid

**Answer (4)**

**Sol.** Vitamin – C is also known as Ascorbic Acid.

## SECTION - B

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. In Carius method for estimation of halogens, 180 mg of an organic compound produced 143.5 mg of AgCl. The percentage composition of chlorine in the compound is _____%.
(Given : molar mass in g $mol^{-1}$ of Ag : 108, Cl : 35.5)

**Answer (20)**

**Sol.** millimoles of AgCl = $\frac{143.5}{143.5} = 1$

Millimoles of Cl = 1

Mass of Cl = $35.5 \times 10^{-3}$ gm

% by mass of Cl = $\frac{35.5}{180} \times 100$

= 19.72%

Nearest integer = 20%

72. Some $CO_2$ gas was kept in a sealed container at a pressure of 1 atm and at 273 K. This entire amount of $CO_2$ gas was later passed through an aqueous solution of $Ca(OH)_2$. The excess unreacted $Ca(OH)_2$ was later neutralized with 0.1 M of 40 mL HCl. If the volume of the sealed container of $CO_2$ was x, then x is _____$cm^3$ (nearest integer).
[Given : The entire amount of $CO_2(g)$ reacted with exactly half the initial amount of $Ca(OH)_2$ present in the aqueous solution]

**Answer (45)**

**Sol.** moles of HCl = $4 \times 10^{-3}$

moles of $Ca(OH)_2$ reacted with HCl = $2 \times 10^{-3}$

Initial amount of $Ca(OH)_2$ = $4 \times 10^{-3}$ moles

Moles of $CO_2$ = $2 \times 10^{-3}$

Volume = $2 \times 10^{-3} \times 22.4$

= $44.8 \times 10^{-3}$ Lit

= 44.8 $cm^3$

Nearest integer = 45

73. The number of molecules/ions that show linear geometry among the following is _____.

$SO_2, BeCl_2, CO_2, N_3^-, NO_2, F_2O, XeF_2, NO_2^+, I_3^-, O_3$

**Answer (6)**

**Sol.** linear geometry is shown by

$BeCl_2; CO_2; N_3^-; XeF_2; NO_2^+; I_3^-$

74. $A \rightarrow B$

The molecule A changes into its isomeric form B by following a first order kinetics at a temperature of 1000 K. If the energy barrier with respect to reactant energy for such isomeric transformation is 191.48 kJ $mol^{-1}$ and the frequency factor is $10^{20}$, the time required for 50% molecules of A to become B is ___ picoseconds (nearest integer). [R = 8.314 J $K^{-1}$ $mol^{-1}$]

**Answer (69)**

**Sol.** $k = Ae^{-\frac{Ea}{RT}}$

$= (10^{20})\left(e^{-\frac{191.48\times1000}{(8.314)(1000)}}\right)$

$= 10^{20}\ (e^{-23.03})$

$= (10^{20})(10^{-10})$

$= 10^{10}$

$t_{1/2} = \frac{0.693}{10^{10}}$

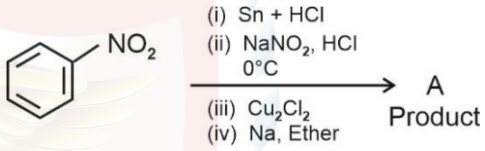
$= 0.693 \times 10^{-10}$

$= 69.3 \times 10^{-12}$

= 69.3 picoseconds

Nearest integer = 69

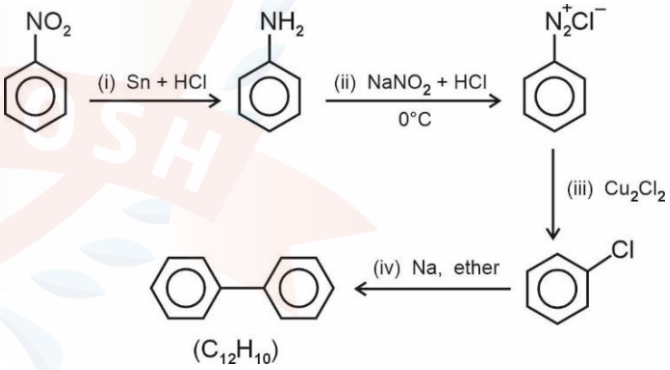
75. Consider the following sequence of reactions :




Molar mass of the product formed (A) is _____ g $mol^{-1}$.

**Answer (154)**

**Sol.**




Molar mass of $A = 154 \frac{gm}{mole}$

❑ ❑ ❑